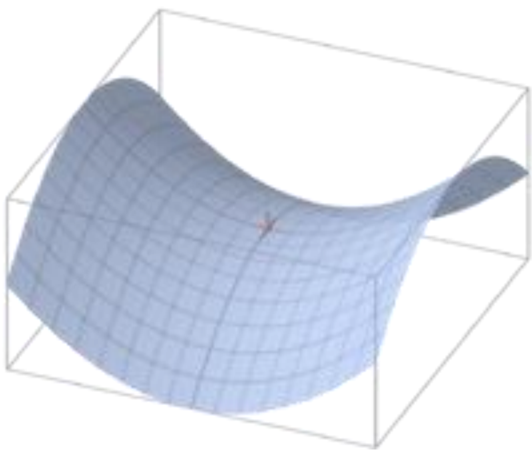
Lecture 10-15
(Problems and discussions with students)

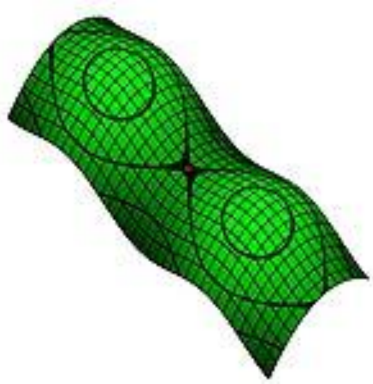
Dimension estimates and model assessments, the resonance effects.

Saddle-point method

In mathematics, a **saddle point** is a point in the domain of a function that is a stationary point but not a local extremum. The name derives from the fact that in two dimensions the surface resembles a saddle that *curves up* in one direction, and *curves down* in a different direction (like a horse saddle or a mountain pass). In terms of contour lines, a saddle point can be recognized, in general, by a contour that appears to intersect itself. For example, two hills separated by a high pass will show up a saddle point, at the top of the pass, like a figure-eight contour line.



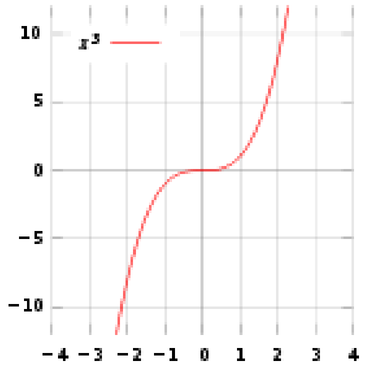
A saddle point on the graph of $z=x^2-y^2$ (in red)



Saddle point between two hills (the intersection of the figure-eight *z*-contour)

Mathematical discussion

In the most general terms, a **saddle point** for a smooth function (whose graph is a curve, surface or hypersurface) is a stationary point such that the curve/surface/etc. in the neighborhood of that point is not entirely on any side of the tangent space at that point.



The plot of $y = x^3$ with a saddle point at 0

In one dimension, a saddle point is a point which is both a stationary point and a point of inflection. Since it is a point of inflection, it is not a local extremum.

In mathematics, the **method of steepest descent** or **stationary phase method** or **saddle-point method** is an extension of Laplace's method for approximating an integral, where one deforms a contour integral in the complex plane to pass near a stationary point (saddle point), in roughly the direction of steepest descent or stationary phase. The saddle-point approximation is used with integrals in the complex plane, whereas Laplace's method is used with real integrals.

The integral to be estimated is often of the form

$$\int_C f(z)e^{\lambda g(z)}dz$$

where C is a contour and λ is large. One version of the method of steepest descent deforms the contour of integration so that it passes through a zero of the derivative $g'(z)$ in such a way that on the contour g is (approximately) real and has a maximum at the zero.

Let us consider the integral

$$I = \int_0^\infty dt\, f(t) e^{g(t)}$$

where $g(t)$ is a function with a sharp peak at $t = t_0 > 0$, but a function $f(t)$ changes slowly near t_0. Then the function $f(t)e^{g(t)}$ can be replaced near the maximum by a simpler function. For this we can represent $g(t)$ near $t \approx t_0$ in the form of expansions:

$$g(t) = g(t_0) + \frac{1}{2}(t - t_0)^2 g''(t_0) + ...$$

Assume that $|g''(t_0)| >> 1/t_0^2$. This is the mathematical definition of a sharp maximum of $g(t)$ at $t = t_0$. Then

$$I \approx f(t_0) \int_{-\infty}^{\infty} dt \exp[g(t_0) - \frac{1}{2}(t - t_0)^2 |g''(t_0)|] = \sqrt{\frac{2\pi}{|g''(t_0)|}} f(t_0) e^{g(t_0)}$$

Saddle-point method is applicable if $g(t_0) >> 1$ which is equivalent to $|g''(t_0)| >> 1/t_0^2$.

Glossary:

a **saddle point** - седловая точка

in two dimensions the surface resembles a saddle - в случае двухмерности поверхность напоминает седло

***curves up, curves down* –** закругление вверх, закругление вниз

a **stationary point** - стационарная точка

a local extremum - локальный экстремум

a contour line – контурная линия

a figure-eight contour line – контурная линия как восьмерка

In the most general terms - В самых общих чертах

a smooth function - гладкая функция

a surface or hypersurface - поверхность или гиперповерхность

a tangent space - пространство касательных кривых

a stationary point and a point of inflection – стационарная точка и точка перегиба

the **method of steepest descent -** метод наискорейшего спуска

the **stationary phase method -** метод стационарной фазы

one deforms a contour integral – делается деформирование контурного интеграла

the complex plane – комплексная плоскость

in roughly the direction of steepest descent - примерно в направлении наискорейшего спуска

the contour of integration passes through a zero of the derivative $g'(z)$ in such a way that on the contour g is (approximately) real and has a maximum at the zero - контур интегрирования проходит через нуль производной $g'(z)$ таким образом, чтобы на этом контуре g был (приблизительно) реальной величиной и имел максимум в нуле (при $z = 0$).

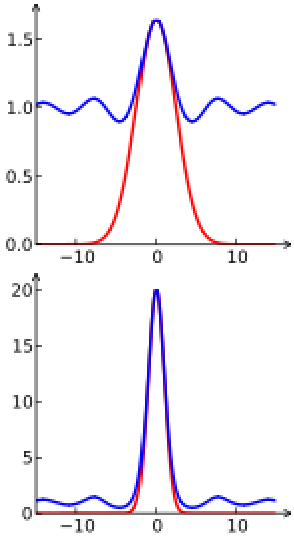
Laplace's method

In mathematics, **Laplace's method**, named after Pierre-Simon Laplace, is a technique used to approximate integrals of the form

$$\int_a^b e^{Mf(x)}\,dx$$

where $f(x)$ is some twice-differentiable function, M is a large number, and the integral endpoints a and b could possibly be infinite.

The idea of Laplace's method



The function $e^{Mf(x)}$, in blue, is shown on top for $M = 0.5$, and at the bottom for $M = 3$. Here, $f(x) = \sin x/x$, with a global maximum at $x_0 = 0$. It is seen that as M grows larger, the approximation of this function by a Gaussian function (shown in red) is getting better. This observation underlies Laplace's method.

Assume that the function $f(x)$ has a unique global maximum at x_0. Then, the value $f(x_0)$ will be larger than other values $f(x)$. If we multiply this function by a large number M, the gap between $Mf(x_0)$ and $Mf(x)$ will only increase, and then it will grow exponentially for the function: $e^{Mf(x)}$.

Thus, significant contributions to the integral of this function will come only from points x in a neighborhood of x_0, which can then be estimated.

General theory of Laplace's method

To state and motivate the method, we need several assumptions. We will assume that x_0 is not an endpoint of the interval of integration, that the values $f(x)$ cannot be very close to $f(x_0)$ unless x is close to x_0, and that the second derivative

$f''(x_0) < 0$.

We can expand $f(x)$ around x_0 by Taylor's theorem,

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2 + R$$

where

$$R = O\left((x - x_0)^3\right).$$

Since f has a global maximum at x_0, and since x_0 is not an endpoint, it is a stationary point, so the derivative of f vanishes at x_0. Therefore, the function $f(x)$ may be approximated to quadratic order

$$f(x) \approx f(x_0) - \frac{1}{2}|f''(x_0)|(x - x_0)^2$$

for x close to x_0 (recall that the second derivative is negative at the global maximum $f(x_0)$).

The assumptions made ensure the accuracy of the approximation

$$\int_a^b e^{Mf(x)}\,dx \approx e^{Mf(x_0)} \int_a^b e^{-M|f''(x_0)|(x-x_0)^2/2}\,dx$$

(see the picture on the right). This latter integral is a Gaussian integral if the limits of integration go from −∞ to +∞ (which can be assumed because the exponential decays very fast away from x_0), and thus it can be calculated. We find

$$\int_a^b e^{Mf(x)}\,dx \approx \sqrt{\frac{2\pi}{M|f''(x_0)|}}e^{Mf(x_0)} \text{ as } M \to \infty.$$

A generalization of this method and extension to arbitrary precision is provided by Fog (2008).

Laplace's method extension: Steepest descent

In extensions of Laplace's method, complex analysis, and in particular Cauchy's integral formula, is used to find a contour *of steepest descent* for an (asymptotically with large M) equivalent integral, expressed as a line integral. In particular, if no point x_0 where the derivative of f vanishes exists on the real line, it may be necessary to deform the integration contour to an optimal one, where the above analysis will be possible. Again the main idea is to reduce, at least asymptotically, the calculation of the given integral to that of a simpler integral that can be explicitly evaluated. See the book of Erdelyi (1956) for a simple discussion (where the method is termed *steepest descents*).

The appropriate formulation for the complex z-plane is

$$\int_a^b e^{Mf(z)}\,dz \approx \sqrt{\frac{2\pi}{-Mf''(z_0)}}e^{Mf(z_0)} \text{ as } M \to \infty.$$

for a path passing through the saddle point at z_0. Note the explicit appearance of a minus sign to indicate the direction of the second derivative: one must *not* take the modulus. Also note that if the integrand is meromorphic, one may have to add residues corresponding to poles traversed while deforming the contour (see for example section 3 of Okounkov's paper *Symmetric functions and random partitions*).

Complex integrals

For complex integrals in the form:

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} g(s)e^{st}\,ds$$

with $t \gg 1$, we make the substitution $t = iu$ and the change of variable $s = c + ix$ to get the Laplace bilateral transform:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} g(c+ix)e^{-ux}e^{icu}\,dx.$$

We then split $g(c+ix)$ in its real and complex part, after which we recover $u = t / i$. This is useful for inverse Laplace transforms, the Perron formula and complex integration.

Example: Stirling's approximation

Laplace's method can be used to derive Stirling's approximation

$$N! \approx \sqrt{2\pi N} N^N e^{-N}$$

for a large integer *N*.

From the definition of the Gamma function, we have

$$N! = \Gamma(N+1) = \int_0^\infty e^{-x} x^N \, dx.$$

Now we change variables, letting

$$x = Nz$$

so that

$$dx = N dz.$$

Plug these values back in to obtain

$$\begin{aligned} N! &= \int_0^\infty e^{-Nz} \, (Nz)^N \, N \, dz \\ &= N^{N+1} \int_0^\infty e^{-Nz} z^N \, dz \\ &= N^{N+1} \int_0^\infty e^{-Nz} e^{N \ln z} \, dz \\ &= N^{N+1} \int_0^\infty e^{N(\ln z - z)} \, dz. \end{aligned}$$

This integral has the form necessary for Laplace's method with

$$f\left(z\right) = \ln z - z$$

which is twice-differentiable:

$$\begin{aligned} f'(z) &= \frac{1}{z} - 1, \\ f''(z) &= -\frac{1}{z^2}. \end{aligned}$$

The maximum of $f(z)$ lies at $z_0 = 1$, and the second derivative of $f(z)$ has at this point the value −1. Therefore, we obtain

$$N! \approx N^{N+1}\sqrt{\frac{2\pi}{N}}e^{-N} = \sqrt{2\pi N}N^N e^{-N}.$$

Model assessments (Модельные оценки)

Evaluation of integrals

1. $\int_0^{\infty} dt \exp(-t^2) \approx \int_0^{x} dt \exp(-t^2) = \int_0^{x} dt \ (1-t^2+t^4/2 \ -...) = x - x^3/3 + x^5/10 - ...$

This assessment is useful for $x \leq 1$.

2. Many integrals can be estimated, highlighting the most significant part in the integrand, for example,

$$I(x) = \int_0^x dt \frac{\exp(t^2)}{\sqrt{x^2 - t^2}} \qquad \textbf{(*)}$$

If $x << 1$, then the exponent in the integrand is $\exp(t^2) \approx 1$. Therefore,

$$I(x) \approx \int_0^x dt \frac{1}{\sqrt{x^2 - t^2}} = \int_0^1 dz \frac{1}{\sqrt{1 - z^2}} = \frac{\pi}{2}$$

If $x >> 1$, the main contribution will make a small area at $\xi = x - t$. Then

$$I(x) = \exp(x^2)\int_0^x d\xi \frac{\exp(-2\xi x + \xi^2)}{\sqrt{2\xi x - \xi^2}} \approx \frac{\exp(x^2)}{2x}\int_0^{\infty} dz \frac{\exp(-z)}{\sqrt{z}} = \frac{\exp(x^2)}{2x}\sqrt{\pi} \qquad \textbf{(**)}$$

where $z = 2\xi x$. At $x \approx 1$ both expressions coincide.

The resonance effects.

In physics, **resonance** is the tendency of a system to oscillate at a greater amplitude at some frequencies than at others. These are known as the system's **resonant frequencies** (or *resonance frequencies*). At these frequencies, even small periodic driving forces can produce large amplitude oscillations, because the system stores vibrational energy.

Resonance occurs when a system is able to store and easily transfer energy between two or more different storage modes (such as kinetic energy and potential energy in the case of a pendulum). However, there are some losses from cycle to cycle, called damping. When damping is small, the resonant frequency is approximately equal to the natural frequency of the system, which is a frequency of unforced vibrations. Some systems have multiple, distinct, resonant frequencies.

Resonance phenomena occur with all types of vibrations or waves: there is mechanical resonance, acoustic resonance, electromagnetic resonance, nuclear magnetic resonance (NMR), electron spin resonance (ESR) and resonance of quantum wave functions. Resonant systems can be used to generate vibrations at a specific frequency (e.g. musical instruments), or pick out specific frequencies from a complex vibration containing many frequencies (e.g. filters).

Resonance was recognized by Galileo Galilei with his investigations of pendulums and musical strings beginning in 1602.[3][4]

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[edit] Examples

Pushing a person in a swing is a common example of resonance. The loaded swing, a pendulum, has a natural frequency of oscillation, its resonant frequency, and resists being pushed at a faster or slower rate.

One familiar example is a playground swing, which acts as a pendulum. Pushing a person in a swing in time with the natural interval of the swing (its resonant frequency) will make the swing go higher and higher (maximum amplitude), while attempts to push the swing at a faster or slower tempo will result in smaller arcs. This is because the energy the swing absorbs is maximized when the pushes are 'in phase' with the swing's oscillations, while some of the swing's energy is actually extracted by the opposing force of the pushes when they are not.

Resonance occurs widely in nature, and is exploited in many man-made devices. It is the mechanism by which virtually all sinusoidal waves and vibrations are generated. Many sounds we hear, such as when hard objects of metal, glass, or wood are struck, are caused by brief resonant vibrations in the object. Light and other short wavelength electromagnetic radiation is produced by resonance on an atomic scale, such as electrons in atoms. Other examples are:

Mechanical and acoustic resonance

- the timekeeping mechanisms of all modern clocks and watches: the balance wheel in a mechanical watch and the quartz crystal in a quartz watch
- the tidal resonance of the Bay of Fundy
- acoustic resonances of musical instruments and human vocal cords
- the shattering of a crystal wineglass when exposed to a musical tone of the right pitch (its resonant frequency)

Electrical resonance

- electrical resonance of tuned circuits in radios and TVs that allow individual stations to be picked up

Optical resonance

- creation of coherent light by optical resonance in a laser cavity

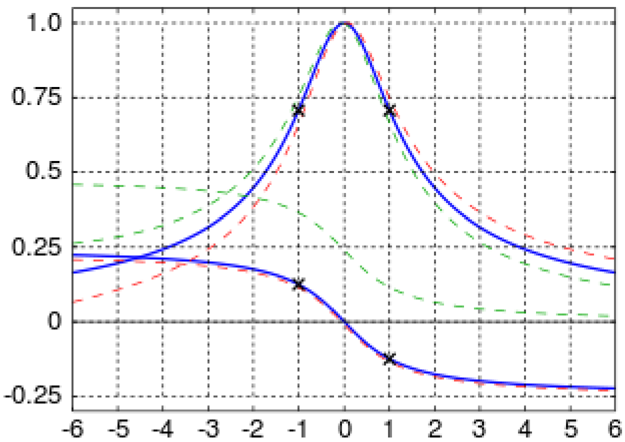
Orbital resonance in astronomy

- orbital resonance as exemplified by some moons of the solar system's gas giants

Atomic, particle, and molecular resonance

- material resonances in atomic scale are the basis of several spectroscopic techniques that are used in condensed matter physics.
  - Nuclear Magnetic Resonance
  - Mössbauer effect
  - Electron Spin Resonance.

[edit] Theory



"Universal Resonance Curve", a symmetric approximation to the normalized response of a resonant circuit; abscissa values are deviation from center frequency, in units of center frequency divided by 2Q; ordinate is relative amplitude, and phase in cycles; dashed curves compare the range of responses of real two-pole circuits for a Q value of 5; for higher Q values, there is less deviation from the universal curve. Crosses mark the edges of the 3-dB bandwidth (gain 0.707, phase shift 45 degrees or 0.125 cycle).

The exact response of a resonance, especially for frequencies far from the resonant frequency, depends on the details of the physical system, and is usually not exactly symmetric about the resonant frequency, as illustrated for the simple harmonic oscillator above. For a lightly damped linear oscillator with a resonant frequency Ω, the *intensity* of oscillations *I* when the system is driven with a driving frequency ω is typically approximated by a formula that is symmetric about the resonant frequency:[5]

$$I(\omega) \propto \frac{\left(\frac{\Gamma}{2}\right)^2}{(\omega - \Omega)^2 + \left(\frac{\Gamma}{2}\right)^2}.$$

The intensity is defined as the square of the amplitude of the oscillations. This is a Lorentzian function, and this response is found in many physical situations involving resonant systems. Γ is a parameter dependent on the damping of the oscillator, and is known as the *linewidth* of the resonance. Heavily damped oscillators tend to have broad linewidths, and respond to a wider range of driving frequencies around the resonant frequency. The linewidth is inversely proportional to the Q factor, which is a measure of the sharpness of the resonance.

In electrical engineering, this approximate symmetric response is known as the *universal resonance curve*, a concept introduced by Frederick E. Terman in 1932 to simplify the approximate analysis of radio circuits with a range of center frequencies and Q values.[6][7]

[edit] Resonators

A physical system can have as many resonant frequencies as it has degrees of freedom; each degree of freedom can vibrate as a harmonic oscillator. Systems with one degree of freedom, such as a mass on a spring, pendulums, balance wheels, and LC tuned circuits have one resonant frequency. Systems with two degrees of freedom, such as coupled pendulums and resonant transformers can have two resonant frequencies. As the number of coupled harmonic oscillators grows, the time it takes to transfer energy from one to the next becomes significant. The vibrations in them begin to travel through the coupled harmonic oscillators in waves, from one oscillator to the next.

Extended objects that experience resonance due to vibrations inside them are called resonators, such as organ pipes, vibrating strings, quartz crystals, microwave cavities, and laser rods. Since these can be viewed as being made of millions of coupled moving parts (such as atoms), they can have millions of resonant frequencies. The vibrations inside them travel as waves, at an approximately constant velocity, bouncing back and forth between the sides of the resonator. If the distance between the sides is d, the length of a round trip is $2d$. In order to cause resonance, the phase of a sinusoidal wave after a round trip has to be equal to the initial phase, so the waves will reinforce. So the condition for resonance in a resonator is that the round trip distance, $2d$, be equal to an integer number of wavelengths $\lambda$of the wave:

$$2d = N\lambda, \qquad N \in \{1, 2, 3, \dots\}$$

If the velocity of a wave is v, the frequency is $f = v/\lambda$so the resonant frequencies are:

$$f = \frac{Nv}{2d} \qquad N \in \{1, 2, 3, \dots\}$$

So the resonant frequencies of resonators, called normal modes, are equally spaced multiples of a lowest frequency called the fundamental frequency. The multiples are often called overtones. There may be several such series of resonant frequencies, corresponding to different modes of vibration.

[edit] Q factor

Main article: Q factor

The **quality factor** or **Q factor** is a dimensionless parameter that describes how under-damped an oscillator or resonator is,[8] or equivalently, characterizes a resonator's bandwidth relative to its center frequency.[9] Higher *Q* indicates a lower rate of energy loss relative to the stored energy of the oscillator, i.e. the oscillations die out more slowly. A pendulum suspended from a high-quality bearing, oscillating in air, has a high *Q*, while a pendulum immersed in oil has a low *Q*. Oscillators with high quality factors have low damping which tends to make them ring longer.

Sinusoidally driven resonators having higher Q factors resonate with greater amplitudes (at the resonant frequency) but have a smaller range of frequencies around the frequency at which they resonate. The range of frequencies at which the oscillator resonates is called the bandwidth. Thus, a high Q tuned circuit in a radio receiver would be more difficult to tune, but would have greater selectivity, it would do a better job of filtering out signals from other stations that lie nearby on the spectrum. High Q oscillators operate over a smaller range of frequencies and are more stable. (See oscillator phase noise.)

The quality factor of oscillators vary substantially from system to system. Systems for which damping is important (such as dampers keeping a door from slamming shut) have $Q = ½$. Clocks, lasers, and other systems that need either strong resonance or high frequency stability need high quality factors. Tuning forks have quality factors around $Q = 1000$. The quality factor of atomic clocks and some high-Q lasers can reach as high as 10^{11}[10] and higher.[11]

There are many alternate quantities used by physicists and engineers to describe how damped an oscillator is that are closely related to its quality factor. Important examples include: the damping ratio, relative bandwidth, linewidth and bandwidth measured in octaves.

[edit] Types of resonance

[edit] Mechanical and acoustic resonance

Main articles: Mechanical resonance, Acoustic resonance, and String resonance

Mechanical resonance is the tendency of a mechanical system to absorb more energy when the frequency of its oscillations matches the system's natural frequency of vibration than it does at other frequencies. It may cause violent swaying motions and even catastrophic failure in improperly constructed structures including bridges, buildings, trains, and aircraft. When designing objects, Engineers must ensure the mechanical resonant frequencies of the component parts do not match driving vibrational frequencies of motors or other oscillating parts, a phenomenon known as resonance disaster.

Avoiding resonance disasters is a major concern in every building, tower and bridge construction project. As a countermeasure, shock mounts can be installed to absorb resonant frequencies and thus dissipate the absorbed energy. The Taipei 101 building relies on a 660-tonne pendulum (730-short-ton) — a tuned mass damper — to cancel resonance. Furthermore, the structure is designed to resonate at a frequency which does not typically occur. Buildings in seismic zones are often constructed to take into account the oscillating frequencies of expected ground motion. In addition, engineers designing objects having engines must ensure that the mechanical resonant frequencies of the component parts do not match driving vibrational frequencies of the motors or other strongly oscillating parts.

Many clocks keep time by mechanical resonance in a balance wheel, pendulum, or quartz crystal

Acoustic resonance is a branch of mechanical resonance that is concerned with the mechanical vibrations across the frequency range of human hearing, in other words sound. For humans, hearing is normally limited to frequencies between about 20 Hz and 20,000 Hz (20 kHz),[12]

Acoustic resonance is an important consideration for instrument builders, as most acoustic instruments use resonators, such as the strings and body of a violin, the length of tube in a flute, and the shape of, and tension on, a drum membrane.

Like mechanical resonance, acoustic resonance can result in catastrophic failure of the object at resonance. The classic example of this is breaking a wine glass with sound at the precise resonant frequency of the glass, although this is difficult in practice.[13]

[edit] Electrical resonance

Main article: Electrical resonance

Electrical resonance occurs in an electric circuit at a particular *resonant frequency* when the impedance of the circuit is at a minimum in a series circuit or at maximum in a parallel circuit (or when the transfer function is at a maximum).

[edit] Optical resonance

Main article: Optical cavity

An optical cavity or **optical resonator** is an arrangement of mirrors that forms a standing wave cavity resonator for light waves. Optical cavities are a major component of lasers, surrounding the gain medium and providing feedback of the laser light. They are also used in optical parametric oscillators and some interferometers. Light confined in the cavity reflects multiple times producing standing waves for certain resonant frequencies. The standing wave patterns produced are called modes. Longitudinal modes differ only in frequency while transverse modes differ for different frequencies and have different intensity patterns across the cross section of the beam. Ring resonators and whispering galleries are examples of optical resonators that do not form standing waves.

Different resonator types are distinguished by the focal lengths of the two mirrors and the distance between them. (Flat mirrors are not often used because of the difficulty of aligning them precisely.) The geometry (resonator type) must be chosen so the beam remains stable, i.e. the beam size does not continue to grow with each reflection. Resonator types are also designed to meet other criteria such as minimum beam waist or having no focal point (and therefore intense light at that point) inside the cavity.

Optical cavities are designed to have a very large Q factor;[14] a beam will reflect a very large number of times with little attenuation. Therefore the frequency line width of the beam is very small compared to the frequency of the laser.

Additional optical resonances are guided-mode resonances and surface plasmon resonance, which result in anomalus reflection and high evanescent fields at resonance. In this case, the resonant modes are guided modes of a waveguide or surface plasmon modes of a dielectric-metallic interface. These modes are usually excited by a subwavelength grating.

[edit] Orbital resonance

Main article: Orbital resonance

In celestial mechanics, an orbital resonance occurs when two orbiting bodies exert a regular, periodic gravitational influence on each other, usually due to their orbital periods being related by a ratio of two small integers. Orbital resonances greatly enhance the mutual gravitational influence of the bodies. In most cases, this results in an *unstable* interaction, in which the bodies exchange momentum and shift orbits until the resonance no longer exists. Under some circumstances, a resonant system can be stable and self correcting, so that the bodies remain in resonance. Examples are the 1:2:4 resonance of Jupiter's moons Ganymede, Europa, and Io, and the 2:3 resonance between Pluto and Neptune. Unstable resonances with Saturn's inner moons give rise to gaps in the rings of Saturn. The special case of 1:1 resonance (between bodies with similar orbital radii) causes large Solar System bodies to clear the neighborhood around their orbits by ejecting nearly everything else around them; this effect is used in the current definition of a planet.

[edit] Atomic, particle, and molecular resonance

Main articles: Nuclear magnetic resonance and Resonance (particle)

21.2 T NMR Magnet at HWB-NMR, Birmingham, UK. In its strong field, the proton resonance is at 900MHz.

Nuclear magnetic resonance (**NMR**) is the name given to a physical resonance phenomenon involving the observation of specific quantum mechanical magnetic properties of an atomic nucleus in the presence of an applied, external magnetic field. Many scientific techniques exploit NMR phenomena to study molecular physics, crystals and non-crystalline materials through

NMR spectroscopy. NMR is also routinely used in advanced medical imaging techniques, such as in magnetic resonance imaging (MRI).

All nuclei containing odd numbers of nucleons have an intrinsic magnetic moment and angular momentum. A key feature of NMR is that the resonant frequency of a particular substance is directly proportional to the strength of the applied magnetic field. It is this feature that is exploited in imaging techniques; if a sample is placed in a non-uniform magnetic field then the resonant frequencies of the sample's nuclei depend on where in the field they are located. Therefore, the particle can be located quite precisely by its resonant frequency.

Electron paramagnetic resonance, otherwise known as **Electron Spin Resonance (ESR)** is a spectroscopic technique similar to NMR, but uses unpaired electrons instead. Materials for which this can be applied are much more limited since the material needs to both have an unpaired spin and be paramagnetic.

The **Mössbauer effect** is the resonant and recoil-free emission and absorption of gamma ray photons by atoms bound in a solid form.

Resonance (particle physics): In quantum mechanics and quantum field theory resonances may appear in similar circumstances to classical physics. However, they can also be thought of as unstable particles, with the formula above valid if the Γ is the decay rate and Ω replaced by the particle's mass M. In that case, the formula comes from the particle's propagator, with its mass replaced by the complex number $M + i\Gamma$. The formula is further related to the particle's decay rate by the optical theorem.

[edit] Failure of the original Tacoma Narrows Bridge

Main article: Tacoma Narrows Bridge (1940)

The dramatically visible, rhythmic twisting that resulted in the 1940 collapse of "Galloping Gertie," the original Tacoma Narrows Bridge, has sometimes been characterized in physics textbooks as a classical example of resonance. However, this description is misleading. The catastrophic vibrations that destroyed the bridge were not due to simple mechanical resonance, but to a more complicated interaction between the bridge and the winds passing through it — a phenomenon known as aeroelastic flutter. Robert H. Scanlan, father of bridge aerodynamics, has written an article about this misunderstanding.[15]

For more details on this topic, see Mechanical resonance.

[edit] Resonance causing a vibration on the International Space Station

The rocket engines for the International Space Station are controlled by autopilot. Ordinarily the uploaded parameters for controlling the engine control system for the Zvezda module will cause the rocket engines to boost the International Space Station to a higher orbit. The rocket engines are hinge-mounted, and ordinarily the operation is not noticed by the crew. But on January 14, 2009, the uploaded parameters caused the autopilot to swing the rocket engines in larger and larger oscillations, at a frequency of 0.5 Hz. These oscillations were captured on video, and lasted for 142 seconds.[16]